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SEPARATION IMPACT OF A CIRCULAR DISK FLOATING ON THE SURFACE OF AN IDEAL INCOMPRESSIBLE FLUID OF INFINITE DEPTH

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The three-dimensional mixed problem of the separation impact of a circular disk floating on the surface of an ideal incompressible unlimited fluid is considered. The position and shape of the contact area between the body and the fluid (and the separation zone) are not known and depend on the relation between the translational and angular velocities acquired by the disk upon impact. Because of this, the problem in question is nonlinear and belongs to the class of free-boundary problems. The problem is solved using the method of Hammerstein-type nonlinear boundary integral equations. This approach allows the fluid flow after impact and the unknown zone of separation of fluid particles to be determined simultaneously.

Key words: *ideal fluid, circular disk, separation impact, separation line.*

Nonlinear problems of hydrodynamic impact were first studied by Sedov [1], who obtained an exact solution of the two-dimensional problem of horizontal impact of a plate. Problems of horizontal and oblique lateral impacts of an ellipse were considered by Kudryavtseva [2] and Korchagin [3, 4]. In the first papers on separation impact, it was noted that ignoring the separation phenomenon in the determination of the point separation and the impulsive forces and momentum leads to large errors. Results of a special experiment supported the main theoretical assumption on the separation of fluid particles from the surface of an ellipse [2]. Numerical analysis of the two-dimensional problem of separation impact of floating bodies was performed by Dvorak and Teselkin [5]. In all studies cited, the fluid was considered ideal, incompressible, homogeneous, and infinite.

Two-dimensional problems of separation impact in the case of a bounded fluid region were considered by Norkin [6], who proposed the following methods for solving the problem of separation impact of a plate: 1) a direct asymptotic method based on the assumption that the tank walls are at large distances from the floating body; 2) a method based on conformal mapping and the mathematical apparatus of dual integral equations. It should be noted that the second method for determining the unknown point of separation of the plate uses the Ogazo variational principle. These methods have been employed to study a number of problems for bounded areas of classical shapes (layer, truncated circular hollow).

The problem of separation impact of an elliptic cylinder floating on the surface of an ideal incompressible fluid of finite depth was studied in [7]. Using the method of Hammerstein-type nonlinear boundary integral equations, this problem is reduced to one nonlinear integral equation, which is solved invoking the method of Krasnosel'skii [8]. We note that this approach allows the velocity potential and the unknown separation zone of fluid particles to be determined simultaneously.

Galanov [9] proposed the method of Hammerstein-type nonlinear integral equations to solve static contact elastic problems with contact areas not known in advance. Detailed reviews of the contact problems solved using this method are given in [10, 11].

A mathematical model for separation impact in an inhomogeneous incompressible fluid was proposed by Yudovich [12]. The important result obtained in [12] is a proof of the existence and uniqueness theorem for the problem of separation impact.

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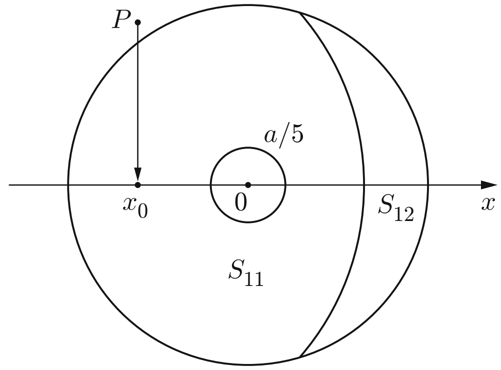

Fig. 1. Formulation of the problem: S_{11} is the zone in which separation of the fluid particles does not occur and S_{12} is the separation zone.

We also note that the problem of rapid entry of a body into water, which is close in mathematical formulation, was considered in [13]. This problem reduces to a variational inequality, which is solved numerically for a body of arbitrary shape.

In all studies listed above, the problem of separation impact was considered in a two-dimensional formulation in which the unknowns are the separation points on the surface of the floating body which separate the regions of continuous impact from the separation zones. A distinctive feature of the present work is that here the unknown is a curve, making the scalar problem infinite dimensional.

1. Formulation of the Problem and Its Reduction to a Hammerstein-Type Nonlinear Integral Equation. We consider the problem of vertical impact of an absolutely hard circular disk floating on the surface of an ideal incompressible fluid occupying the entire lower half-space. It is assumed that the impact results in separation of fluid particles from the surface of the disk (separation impact). According to [1, 12], the potential Φ of the velocities acquired by the fluid particles upon the impact is determined by solving the mixed boundary-value potential problem with the a priori unknown contact area:

$$\Delta\Phi = 0,$$

$$\frac{\partial\Phi}{\partial z} = V_n, \quad \Phi \leqslant 0, \quad r \in S_{11},$$

$$\frac{\partial\Phi}{\partial z} \geqslant V_n, \quad \Phi = 0, \quad r \in S_{12}, \tag{1.1}$$

$$\Phi = 0, \qquad r \in S_2,$$

$$\Phi \to 0, \qquad r \to \infty;$$

$$V_n = v_0 - \omega x;$$

$$\int\limits_D (\nabla\Phi)^2 dD < \infty. \tag{1.2}$$

Here v_0 and ω are the translational and angular velocities acquired by the disk upon impact, $S_1 = \{z = 0, 0 < r < a\} = S_{11} \cup S_{12}$ is the surface of the disk, S_{11} is the part of the boundary on which separation of the fluid particles does not occur, S_{12} is the separation zone, $S_2 = \{z = 0,\ a < r < \infty\}$ is the free fluid surface, a is the radius of the disk, and $D = \{-\infty < x, y < \infty,\ z > 0\}$ is the region occupied by the fluid (Fig. 1). The Cartesian coordinates x, y, and z are introduced in such a manner that the x and y axes are located in the plane of the fluid free surface, the z axis is directed vertically downward into the fluid depth; the coordinate origin is at the center of the disk. Without loss of generality, we assume that the disk rotates around the y axis ($\omega > 0$).

Condition (1.2), which postulates the finiteness of the kinetic energy of the fluid flow after the impact closes the mathematical formulation of the problem. The existence and uniqueness theorem for the solution of the separation impact problem was proved in [12].

To solve the problem completely, it is also necessary to take into account the equations of momentum and angular momentum of the circular disk during impact. These equations define the relationship between the external impact momentum $P = (0, 0, P_z)$ and the point of its application $R_0 = (x_0, 0, 0)$, on the one hand, and the translational and angular velocities v_0 and ω acquired by the disk upon impact, on the other hand. If the mass and momentum of inertia of the disk are ignored, the above equations lead to the relations

$$I = -P_z, \qquad M = x_0 P_z, \qquad I = \rho \iint\limits_{S_1} \Phi\, ds, \qquad M = -\rho \iint\limits_{S_1} x\Phi\, ds, \tag{1.3}$$

where I and M are the momentum and angular momentum acquired by the floating disk as a result of impact from the fluid and ρ is the density of the fluid.

The problem is complicated by the fact that the separation line on the disk surface which separates the region of continuous impact from the separation zone is not known in advance and is to be determined during the solution of the problem. Therefore, the problem is nonlinear and belongs to the class of free-boundary problems.

We note that the condition of separation of the fluid particles from the disk surface (as well as the condition of separationless impact) is determined exactly by solving the corresponding linear problem of continuous impact of a circular disk. According to [14], this condition is written as

$$3v_0/(2\omega a) < 1.$$

The above inequality implies that the point of application of the external impact momentum is outside a circle of radius $a/5$ with center at the center of the disk (see Fig. 1). Otherwise, the impact occurs without separation.

The problem is solved using the method of Hammerstein-type nonlinear boundary integral equations [7–11]. This approach allows one to simultaneously determine the velocity potential and the unknown line on the disk surface separating the region of continuous impact from the separation zone. This method has been employed [7] to solve some two-dimensional problems of hydrodynamic impact. According to [7], problem (1.1), (1.2) can be reduced to a nonlinear integral equation.

Along with the initial nonlinear problem, we consider the linear mixed boundary-value problem in half-space with a circular region of separation of the boundary conditions:

$$\Delta W = 0, \qquad \Big(\frac{\partial W}{\partial z}\Big)_{S_1} = f(x, y), \qquad (W)_{S_2} = 0, \qquad (W)_\infty = 0. \tag{1.4}$$

Here $f(x, y)$ is a continuous function which is even in the variable y.

Below, an important role is played by a linear operator K which specifies a correspondence between the normal derivative of the function W on S_1 and the solution of problem (1.4) on S_1:

$$K(f) = W. \tag{1.5}$$

In view of (1.5), the problem of separation impact of a circular disk reduces to the following system of unilateral constraints for the new function u defined on S_1:

$$u = \frac{\partial \Phi}{\partial n} - V_n;$$

$$\begin{aligned} -K(u) &\geqslant K(V_n), \qquad u = 0, \qquad r \in S_{11}, \\ -K(u) &= K(V_n), \qquad u \geqslant 0, \qquad r \in S_{12}. \end{aligned} \tag{1.6}$$

The separation line can be determined from the condition contained in system (1.6) which can be reduced to one nonlinear integral equation of the form [7–11]

$$Tv = 0, \qquad Tv = \mu v^- - K(v^+) - K(V_n). \tag{1.7}$$

Here μ is an arbitrary positive parameter and the nonlinear operators v^- and v^+ are given by the relations

$$v^- = v^-(M) = \inf\{v(M), 0\}, \qquad v^+ = v^+(M) = \sup\{v(M), 0\}, \qquad v(M) = v^-(M) + v^+(M).$$

The relationship between the solutions of system (1.6) and the nonlinear equation (1.7) is written as

$$u = (v^*)^+, \qquad S_{12} = \{(x, y) \in S_1 \colon v^*(x, y) \geqslant 0\}.$$

Thus, to solve system (1.6), it is sufficient to find the solution v^* of the nonlinear equation (1.7). In this case, the a priori unknown separation zone S_{12} is defined as a set of all points $(x, y) \in S_1$ that satisfy the inequality $v^*(x, y) \geqslant 0$. It is of interest to determine the separation line between the sets S_{12} and $S_{11} = S_1 \setminus S_{12}$.

Once Eq. (1.7) is solved, the velocity potential Φ on the boundary S_1 can be found by the formula

$$\Phi = K((v^*)^+ + V_n).$$

2. Construction of the Operator *K*. According to [15], we seek a solution of problem (1.4) in the form of a Fourier cosine series on the angular coordinate φ in the cylindrical coordinates r, φ, and z:

$$W(r, \varphi, z) = \sum_{n=0}^{\infty} \cos n\varphi \int_0^{\infty} A_n(\lambda)\, \mathrm{e}^{-\lambda z}\, J_n(\lambda r)\, d\lambda. \tag{2.1}$$

Here $A_n(\lambda)$ are arbitrary functions and $J_n(\lambda r)$ are Bessel functions. Thus, the Laplace equation and the condition at infinity will be satisfied. Next, satisfying the mixed boundary-value conditions at $z = 0$, we obtain the following system of dual equations for the unknown functions $A_n(\lambda)$:

$$\sum_{n=0}^{\infty} \cos n\varphi \int_0^{\infty} \lambda A_n(\lambda) J_n(\lambda r)\, d\lambda = -f_0(r, \varphi), \qquad 0 < r < a; \tag{2.2}$$

$$\sum_{n=0}^{\infty} \cos n\varphi \int_0^{\infty} A_n(\lambda) J_n(\lambda r)\, d\lambda = 0, \qquad a < r < \infty. \tag{2.3}$$

Here $f_0(r, \varphi) = f(r \cos\varphi, r \sin\varphi)$. System (2.2), (2.3) is solved using discontinuous integrals of the form [15]

$$\int_0^{\infty} \sqrt{\lambda}\, J_{n+1/2}(\lambda t) J_n(\lambda r)\, d\lambda = \begin{cases} 0, & 0 \leqslant t < r, \\ \dfrac{\sqrt{2}\, r^n}{\sqrt{\pi}\, t^{n+1/2} \sqrt{t^2 - r^2}}, & 0 < r < t; \end{cases} \tag{2.4}$$

$$\int_0^{\infty} \sqrt{\lambda} J_{n+1}(\lambda r) J_{n+1/2}(\lambda t)\, d\lambda = \begin{cases} 0, & 0 < r < t, \\ \dfrac{\sqrt{2}\, t^{n+1/2}}{\sqrt{\pi}\, r^{n+1} \sqrt{r^2 - t^2}}, & r > t. \end{cases} \tag{2.5}$$

Setting

$$A_n(\lambda) = \sqrt{\frac{\pi\lambda}{2}} \int_0^{a} \sqrt{t}\, \varphi_n(t) J_{n+1/2}(\lambda t)\, dt \tag{2.6}$$

and taking into account (2.4), it is possible to satisfy the second equation of system (2.3) for any continuous functions $\varphi_n(t)$. Next, finding the coefficients of expansion of the function $f_0(r, \varphi)$ in a Fourier series in cosines, from (2.6) we obtain the equality

$$\sqrt{\frac{\pi}{2}} \int_0^{\infty} \lambda^{3/2} J_n(\lambda r)\, d\lambda \int_0^{a} \sqrt{t}\, \varphi_n(t) J_{n+1/2}(\lambda t)\, dt = F_n(r), \qquad 0 < r < a,$$

$$F_0(r) = -\frac{1}{\pi} \int_0^{\pi} f_0(r, \varphi)\, d\varphi, \qquad F_n(r) = -\frac{2}{\pi} \int_0^{\pi} f_0(r, \varphi) \cos n\varphi\, d\varphi, \quad n = 1, 2, 3, \dots .$$

Using the known relation [15]

$$\lambda J_n(\lambda r) = r^{-n-1} [r^{n+1} J_{n+1}(\lambda r)]'_r,$$

we write the last equation as

$$\sqrt{\frac{\pi}{2}} \frac{d}{dr} \int_0^{\infty} r^{n+1} J_{n+1}(\lambda r) \sqrt{\lambda}\, d\lambda \int_0^{a} \sqrt{t}\, \varphi_n(t) J_{n+1/2}(\lambda t)\, dt = r^{n+1} F_n(r).$$

Integrating the above equality in the variable r, changing the order of integration, and using the discontinuous integral (2.5), we obtain the Schlomilch integral equation

$$\int_0^r \frac{t^{n+1}\varphi_n(t)\,dt}{\sqrt{r^2-t^2}} = \int_0^r F_n(\rho)\rho^{n+1}\,d\rho,$$

whose solution has the form

$$t^{n+1}\varphi_n(t) = \frac{2}{\pi}\frac{d}{dt}\int_0^t \frac{r\,dr}{\sqrt{t^2-r^2}} \int_0^r F_n(\rho)\rho^{n+1}\,d\rho.$$

Integrating this solution in parts and bringing the derivative under the integral sign, we obtain the following expression for the function $\varphi_n(t)$:

$$t^n\varphi_n(t) = \frac{2}{\pi}\int_0^t \frac{r^{n+1}F_n(r)\,dr}{\sqrt{t^2-r^2}}. \tag{2.7}$$

Thus, the solution of problem (1.4) are formulas (2.1), (2.6), and (2.7). To construct the operator K, it remains to find the function W on the disk surface. Substituting (2.6) into (2.1) and using the discontinuous integral (2.4), we have

$$W(r,\varphi,0) = \sum_{n=0}^{\infty}\cos n\varphi \int_r^a \frac{r^n\varphi_n(t)\,dt}{t^n\sqrt{t^2-r^2}}.$$

Expressing the function $\varphi_n(t)$ in terms of the integrals containing the function $f_0(r,\varphi)$, we obtain the equality

$$W(r,\varphi,0) = -\frac{2}{\pi^2}\int_r^a \frac{dt}{\sqrt{t^2-r^2}} \int_0^t \frac{s\,ds}{\sqrt{t^2-s^2}} \int_0^\pi f_0(s,\theta)\,d\theta$$

$$-\frac{4}{\pi^2}\int_r^a \frac{dt}{\sqrt{t^2-r^2}} \int_0^t \frac{s\,ds}{\sqrt{t^2-s^2}} \int_0^\pi f_0(s,\theta) \sum_{n=1}^{\infty}\left(\frac{rs}{t^2}\right)^n \cos n\varphi\ \cos n\theta\,d\theta.$$

After transformation of the product of the cosines to their sum and summation of the series [16]

$$\sum_{n=1}^{\infty}\left(\frac{rs}{t^2}\right)^n[\cos n(\varphi-\theta)+\cos n(\varphi+\theta)]$$

$$= \frac{t^4-t^2rs\cos(\varphi-\theta)}{t^4-2t^2rs\cos(\varphi-\theta)+r^2s^2} + \frac{t^4-t^2rs\cos(\varphi+\theta)}{t^4-2t^2rs\cos(\varphi+\theta)+r^2s^2} - 2,$$

the above equality becomes

$$W(r,\varphi,0) = -\frac{2}{\pi^2}\int_r^a \frac{dt}{\sqrt{t^2-r^2}} \int_0^t \frac{s\,ds}{\sqrt{t^2-s^2}} \int_0^\pi f_0(s,\theta)$$

$$\times\left[\frac{t^4-t^2rs\cos(\varphi-\theta)}{t^4-2t^2rs\cos(\varphi-\theta)+r^2s^2} + \frac{t^4-t^2rs\cos(\varphi+\theta)}{t^4-2t^2rs\cos(\varphi+\theta)+r^2s^2} - 1\right]d\theta.$$

From this, changing the order of integration over the variables t and s and transforming the expression in square brackets by means of some elementary operations, we obtain the following representation of the operator K:

$$K(f)(r,\varphi) = W(r,\varphi,0) = \frac{1}{\pi^2}\int_0^\pi\int_0^a sf_0(s,\theta)K(r,\varphi;s,\theta)\,ds\,d\theta,$$

$$K(r,\varphi;s,\theta)=\int\limits_p^a \frac{r^2s^2-t^4}{\sqrt{(t^2-r^2)(t^2-s^2)}}\,K_1(t,r,s,\varphi,\theta)\,dt,$$

$$K_1(t,r,s,\varphi,\theta)=\frac{1}{t^4-2t^2rs\cos(\varphi-\theta)+r^2s^2}+\frac{1}{t^4-2t^2rs\cos(\varphi+\theta)+r^2s^2}.$$

Here $p=r$ at $s<r$ and $p=s$ at $s>r$. In the case $s=r$, $\varphi\neq\theta$, the expression for the kernel is obtained by passing to the limit as $s\to r$.

3. Numerical Implementation and Analysis of Results. The nonlinear integral equation (1.7) is solved numerically using a modified Newton–Kantorovich method:

$$v_{n+1}=v_n-(T'v_0)^{-1}Tv_n,\qquad n=0,1,2,3,\ldots\ .$$

Here as the initial approximation, we use the function $v_0(x)\equiv -c$ ($c=\text{const}$, $c>0$). In this case, the Frechet derivative $T'v_0$ of the operator T: $C(S_1)\to C(S_1)$ exists and is a linear operator of the form

$$T'v_0=\mu E,$$

where E is a unit operator. As a result, the solution of the nonlinear integral equation (1.7) reduces to the explicit iterative process

$$v_{n+1}=v_n-\mu^{-1}Tv_n,\qquad n=0,1,2,3,\ldots\ . \tag{3.1}$$

The algorithm of this method allows one to simultaneously find the velocity potential Φ and the a priori unknown separation line between the zone of separationless impact S_{11} and the separation zone S_{12}. In the discretization of the equation, the symmetry of the solution v^* about the x axis is taken into account and only the upper half of the disk is considered. The singularity in the kernel of the integral operator K is smoothed out by the formulas

$$f_0(s,\theta)=[f_0(s,\theta)-f_0(r,\theta)]+[f_0(r,\theta)-f_0(r,\varphi)]+f_0(r,\varphi),$$

$$K(f)(r,\varphi)=\frac{1}{\pi^2}\int\limits_0^a\int\limits_0^\pi s[f_0(s,\theta)-f_0(r,\theta)]K(r,\varphi;s,\theta)\,d\theta\,ds$$

$$+\frac{1}{\pi^2}\int\limits_0^\pi[f_0(r,\theta)-f_0(r,\varphi)]\int\limits_0^a sK(r,\varphi;s,\theta)\,ds\,d\theta-\frac{2}{\pi}\,f_0(r,\varphi)\sqrt{a^2-r^2}.$$

We note that, for any fixed $s\neq r$, the internal integral over the angular coordinate θ has no singularity and tends to zero as $s\to r$. The other internal integral over the variable s for any fixed $\theta\neq\varphi$ also has no singularity, and as $\theta\to\varphi$, this integral, as a function of the parameters θ and φ, has a weak logarithmic singularity which is smoothed out by the difference $f_0(r,\theta)-f_0(r,\varphi)$. The repeated integrals are calculated by the midpoint rule.

During the numerical implementation of the method of nonlinear integral equations, the problem of determining the kernel $K(r,\varphi;s,\theta)$ is solved, which is not connected to the iterative process (3.1). After transformation to polar coordinates and the change of variables $\varphi\to\pi\varphi$, $r\to ar$, a uniform grid is introduced: $\varphi_m=(2m-1)/(2m)$, $r_k=(2k-1)/(2k)$, $m,k=1,\ldots,n$. After the completion of the iterative process, an approximate solution $v^*(\varphi,r)$ of the nonlinear integral equation (1.7) is found, which is defined at the grid nodes. This solution is negative in the zone of separationless impact S_{11} and is positive in the separation zone S_{12}. To determine the separation line on the disk surface which separates the zone of separationless impact from the separation zone, we fix the angular coordinate $\varphi=\varphi_m$ and find a value of r for which the sign of the function $v^*(\varphi_m,r)$ changes. After that, in the vicinity of the presumed root, we construct a Lagrangian interpolation polynomial, whose root is determined by the bisection method. In this way, we obtain separation points that correspond to the nodes φ_m ($m=1,2,\ldots,n$), i.e., a parametric equation of the separation line in the polar coordinates $r=r_*(\varphi)$ defined at the nodes φ_m. After that, the continuous curve $r=r_*(\varphi)$ can be reconstructed by graphic interpolation. We note that the calculation accuracy depends mainly on the number of partitioning points n in the interval $[0;1]$ and the number of iterations p. In the calculations, we set $n=94$, $p=200$, and $\mu=1$.

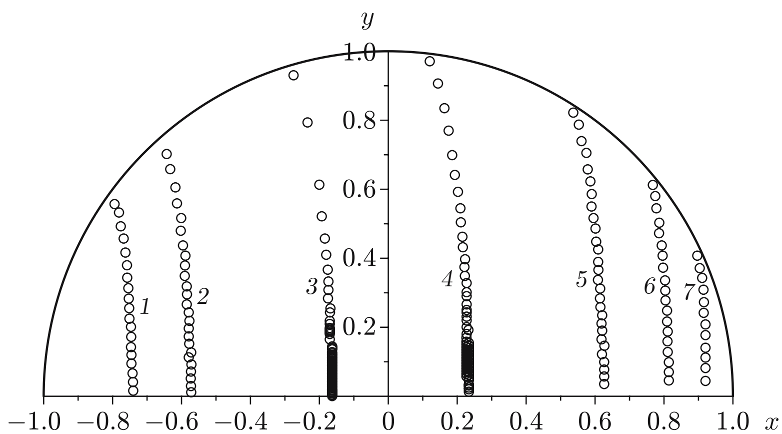

Fig. 2. Upper halves of the disk separation lines for $\alpha = -0.8$ (1), $-2/3$ (2), $-1/3$ (3), 0 (4), 1/3 (5), 0.5 (6), and 0.6 (7).

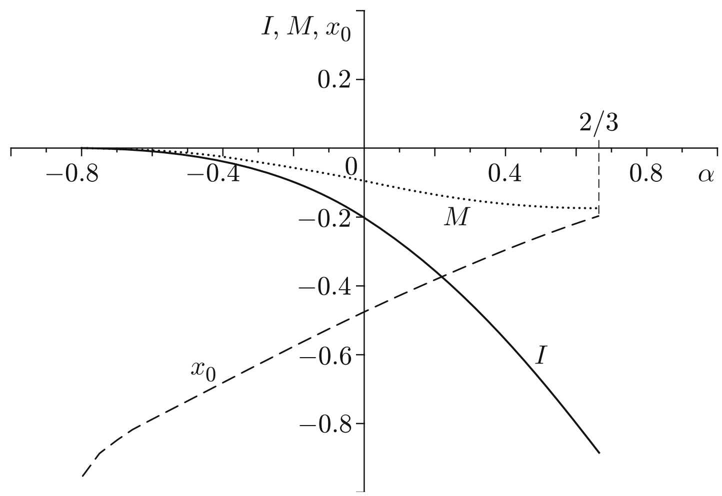

Fig. 3. Values of I, M, and x_0 versus parameter α.

Transformation to dimensionless variables was performed by the formulas

$$x \to ax, \qquad y \to ay, \qquad z \to az,$$

$$x_0 \to ax_0, \qquad \Phi \to \omega a^2 \Phi, \qquad f \to \omega a f,$$

$$K \to aK, \qquad I \to \rho\omega a^4 I, \qquad M \to \rho\omega a^5 M.$$

We note that, formally, the dimensionless quantities are obtained by setting $a = 1$, $\omega = 1$, and $\rho = 1$.

Figure 2 gives the upper halves of the separation lines for various values of $\alpha = v_0/(\omega a)$. Table 1 lists the dimensionless components of the total impact momentum and angular momentum acquired by the floating disk upon impact by the fluid and the corresponding dimensionless coordinates of the points of application of the external impact momentum. The dependences of the quantities I, M, and x_0 on the parameter α are illustrated in Fig. 3.

We note that, for $\alpha \to 2/3$, the obtained results are in good agreement with the well-known exact solution of the problem of separationless impact of a circular disk (case $\alpha \geqslant 2/3$) [14]. In this case, for the dimensionless quantities I and M, the following the relations hold:

$$I = -4\alpha/3, \qquad M = -8/45.$$

TABLE 1

Values of Components of Total Impact Momentum and Angular Momentum

α	I	M	x_0	α	I	M	x_0
-0.80	-0.001	-0.001	-0.954	0.50	-0.674	-0.174	-0.258
$-2/3$	-0.006	-0.005	-0.828	0.60	-0.801	-0.177	-0.222
$-1/3$	-0.056	-0.036	-0.645	0.66	-0.881	-0.178	-0.202
0	-0.203	-0.097	-0.478	2/3	-0.889	-0.178	-0.200
1/3	-0.484	-0.157	-0.326				

TABLE 2

Coordinates of the Separation Point Determined by Formula (3.2) and by Solving Problem (1.1)

φ	$r(\varphi)$	$r_*(\varphi)$	φ	$r(\varphi)$	$r_*(\varphi)$
2.557	0.942	0.946	2.958	0.761	0.752
2.690	0.855	0.852	3.091	0.743	0.735
2.824	0.794	0.789			

For $\alpha < 2/3$, this problem becomes nonlinear and extremely complicated and can be solved only using the proposed method.

We determine the separation line in the limiting case $\alpha \to -1$, which corresponds to a strong separation of the disk. Transforming to cylindrical coordinates in problem (1.1), we fix the polar angle φ and consider the section $r > 0$, $z > 0$. Then, moving the coordinate origin to the middle of the contact area (at the midpoint $[r_*(\varphi), a]$), carrying out stretching of the coordinates, and introducing a small parameter that characterizes the dimension of the contact area

$$r \to \frac{a - r_*}{2} r + \frac{a + r_*}{2}, \qquad z \to \frac{a - r_*}{2} z, \qquad \Phi \to \frac{a - r_*}{2} \Phi,$$

we pass to the limit as $r_* \to a$. As a result, we have a two-dimensional problem which, in essence, coincides with the problem of separation impact of a plate. Using the explicit expression for the plate separation point, we find the main term of the asymptotics of the disk separation line:

$$r_*(\varphi) = \frac{4}{3\cos\varphi} \frac{v_0}{\omega a} - \frac{1}{3}. \tag{3.2}$$

A comparison of the numerical results obtained by the asymptotic formula (3.2) with the corresponding results found using the method of nonlinear integral equations shows that they are in good agreement over a range of values of the characteristic parameter of the problem α. Table 2 gives the results of comparison for the case $\alpha = -0.8$ (the relative error is not more than 1.2%). As the value $\alpha = -1$ is approached, the accuracy of calculations using the asymptotic formula (3.2) increases. It is should be noted that, even for $\alpha = -2/3$, the relative error is not more than 3.5%.

In the solution of Eq. (1.7), it is assumed that the velocities acquired by the disk upon impact are known and the components of the total impact momentum and angular momentum are found as integrals containing the velocity potential Φ. Next, the external impact momentum P_z and the point of its application x_0 are determined by formulas (1.3).

We consider a different formulation of the problem in which the quantities P_z and x_0 are specified. We note that the position of the separation line does not depend on the momentum of the external impact force and depends only on the point of its application x_0. Setting x_0 and using the determining relation (1.3), we obtain the following nonlinear equation for the quantity $\alpha = v_0/(\omega a)$:

$$F(\alpha) = 0, \qquad F(\alpha) = x_0 + M/I.$$

This equation is solved using the method of secants. It is should be noted that the dependence $F(\alpha)$ is defined by the system of unilateral constraints (1.6). Therefore, in each step of the iterative process considered, it is necessary to solve the nonlinear integral equation (1.7). Numerical calculations show that the method of secants converges rapidly. Let, for example, $x_0 = -0.3$. Then, setting the initial approximations $\alpha_0 = 0.6$ and $\alpha_1 = 0.62$, we obtain the following values of α: $\alpha_2 = 0.36669$, $\alpha_3 = 0.39688$, $\alpha_4 = 0.39434$, $\alpha_5 = 0.39432$, and $\alpha_6 = 0.39432$. Table 3 gives values of the parameter α that correspond to various values of x_0.

TABLE 3

Parameter α versus Coordinates of Point of Application of Impact Momentum

x_0	α	x_0	α
-0.25	0.521	-0.70	-0.438
-0.30	0.394	-0.80	-0.620
-0.40	0.166	-0.90	-0.762
-0.50	-0.044		

It should also be noted that ignoring the mass and the moment of inertia of the disk in the determination of the quantity $\alpha = \alpha(x_0)$ leads to an error of order $O(h)$, $h \to 0$ (h is the thickness of the disk).

Conclusions. The spatial problem of separation impact of a circular disk floating on the surface of an ideal incompressible fluid of infinite depth was studied using the method of Hammerstein-type nonlinear integral equations. The effect of various modes of motion of the disk on the shape and position of the separation line on the disk surface which separates the zone of separationless impact from the separation zone was studied. An algorithm for determining the separation line from the specified point of application of the external impact momentum was proposed. In addition, the components of the total impact momentum and angular momentum acquired by the floating disk upon impact were obtained.

We note that the more general problem of separation impact of a solid body floating on the surface of an ideal incompressible fluid in a bounded tank can also be reduced to a Hammerstein-type nonlinear integral equation. In this case, the main difficulty is in constructing and approximating the operator K of the corresponding linear mixed boundary-value problem.

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